

# Heavy-quark expansion for D and B mesons in nuclear matter

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**Abstract.** The planned experiments at FAIR enable the study of medium modifications of  $D$  and  $B$  mesons in (dense) nuclear matter. Evaluating QCD sum rules as a theoretical prerequisite for such investigations encounters heavy-light four-quark condensates. We utilize an extended heavy-quark expansion to cope with the condensation of heavy quarks.

## 1 Introduction

The forthcoming experimental perspectives for in-medium heavy-light quark meson spectroscopy, in particular at FAIR, are accompanied by the need for sophisticated theoretical analyses, e. g. [1–6]. When utilizing QCD sum rules [7–9], this requires a thorough discussion of heavy-quark condensates in general, and, in particular, in the nuclear medium [10]. Therefore, the heavy-quark expansion (HQE), originally developed for the heavy two-quark condensate  $\langle \bar{Q}Q \rangle$  in vacuum, is extended to four-quark condensates and to the in-medium case, thus going beyond previous approaches, e. g. [11]. Specific formulas are derived and presented.

## 2 Recollection: HQE in vacuum

In [12], a general method is introduced for vacuum condensates involving heavy quarks  $Q$  with mass  $m_Q$ . The heavy-quark condensate is considered as the one-point function

$$\langle 0 | \bar{Q}Q | 0 \rangle = -i \int \frac{d^4p}{(2\pi)^4} \langle 0 | \text{Tr}_{c,D} S_Q(p) | 0 \rangle \quad (1)$$

expressed by the heavy-quark propagator  $S_Q$  in a weak classical gluonic background field in Fock-Schwinger gauge,  $S_Q(p) = \sum_{k=0}^{\infty} S_Q^{(k)}(p)$  with  $S_Q^{(k)}(p) = (-1)^k S_Q^{(0)}(p) \gamma^{\mu_1} \tilde{A}_{\mu_1} S_Q^{(0)}(p) \dots \gamma^{\mu_k} \tilde{A}_{\mu_k} S_Q^{(0)}(p)$ , incorporating the free heavy-quark propagator  $S_Q^{(0)}(p) = (\gamma^\mu p_\mu + m_Q)/(p^2 - m_Q^2)$  and the derivative operator  $\tilde{A}$  emerging from a Fourier transform defined as  $\tilde{A}_\mu = \sum_{m=0}^{\infty} \tilde{A}_\mu^{(m)}$  with  $\tilde{A}_\mu^{(m)} = -\frac{(-i)^{m+1}g}{m!(m+2)} (D_{\alpha_1} \dots D_{\alpha_m} G_{\mu\nu}(x))_{x=0} \partial^\nu \partial^{\alpha_1} \dots \partial^{\alpha_m}$  [13, 14]. In this way, the heavy-quark propagator interacts with the complex QCD ground state via soft gluons generating a series expansion in the inverse heavy-quark mass. The compact notation (1) differs from [12], but provides a comprehensive scheme

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easily extendable to in-medium condensates. The first HQE terms of the heavy two-quark condensate (1) reproduce [12]:

$$\begin{aligned} \langle 0 | \bar{Q} Q | 0 \rangle &= -\frac{g^2}{48\pi^2 m_Q} \langle G^2 \rangle - \frac{g^3}{1440\pi^2 m_Q^3} \langle G^3 \rangle - \frac{g^4}{120\pi^2 m_Q^3} \langle (DG)^2 \rangle + \dots \\ &= \text{[Diagram: Heavy quark loop with gluon condensate]} + \left( \text{[Diagram: Heavy quark loop with gluon condensate]} + \text{[Diagram: Heavy quark loop with gluon condensate]} \right) + \text{[Diagram: Heavy quark loop with gluon condensate]} + \dots \end{aligned} \quad (2)$$

with the notation

$$\langle G^2 \rangle = \langle 0 | G_{\mu\nu}^A G^{A\mu\nu} | 0 \rangle, \quad (3)$$

$$\langle G^3 \rangle = \langle 0 | f^{ABC} G_{\mu\nu}^A G^{B\nu}{}_\lambda G^{C\lambda\mu} | 0 \rangle, \quad (4)$$

$$\langle (DG)^2 \rangle = \langle 0 | \left( \sum_f \bar{q}_f \gamma_\mu t^A q_f \right)^2 | 0 \rangle. \quad (5)$$

The graphic interpretation of the terms in (2) is depicted too: the solid lines denote the free heavy-quark propagators and the curly lines are for soft gluons whose condensation is symbolized by the crosses, whereas the heavy quark-condensate is symbolized by the crossed circles [15]. An analogous expression for the mixed heavy-quark gluon condensate can be obtained along those lines which contains, however, a term proportional to  $m_Q$ . The leading-order term in (2) was employed already in [7] in evaluating the sum rule for charmonia.

The vacuum HQE method was rendered free of UV divergent results for higher mass-dimension heavy-quark condensates by requiring at least one condensing gluon per condensed heavy-quark [15, 16], which prevents unphysical results, where the condensation probability of heavy-quark condensates rises for an increasing heavy-quark mass.

### 3 Application of HQE to in-medium heavy-light four-quark condensates

The above method can be extended to in-medium situations. Our approach contains two new aspects: (i) formulas analogous to equation (1) are to be derived for heavy-quark condensates, e. g.  $\langle \bar{Q} \not{D} Q \rangle$ ,  $\langle \bar{Q} \not{D} \sigma G Q \rangle$ ,  $\langle \bar{q} \not{t}^A q \bar{Q} \not{t}^A Q \rangle$ , which additionally contribute to the in-medium operator product expansion (OPE) and (ii) medium-specific gluonic condensates, e. g.  $\langle G^2/4 - (vG)^2/v^2 \rangle$ ,  $\langle G^3/4 - f^{ABC} G_{\mu\nu}^A G^{B\nu}{}_\lambda G^{C\lambda\mu} v^\mu v_\nu / v^2 \rangle$ , enter the HQE of heavy-quark condensates for both, vacuum and additional medium condensates, where  $\langle \dots \rangle$  denotes Gibbs averaging.

We are especially interested in heavy-light four-quark condensates entering the OPE of  $D$  and  $B$  mesons, inter alia, in terms corresponding to the next-to-leading-order perturbative diagrams with one light-quark ( $q$ ) and one heavy-quark ( $Q$ ) line cut. There are 24 two-flavour four-quark condensates in the nuclear medium [17] represented here in a compact notation by  $\langle \bar{q} \Gamma T^A q \bar{Q} \Gamma' T^A Q \rangle$ , where  $\Gamma$  and  $\Gamma'$  denote Dirac structures and  $T^A$  with  $A = 0, \dots, 8$  are the generators of  $SU(3)$  supplemented by the unit element ( $A = 0$ ). We obtain the analogous formula to (1) for heavy-light four-quark condensates:

$$\langle \bar{q} \Gamma T^A q \bar{Q} \Gamma' T^A Q \rangle = -i \int \frac{d^4 p}{(2\pi)^4} \langle \bar{q} \Gamma T^A q \text{Tr}_{c,D} [\Gamma' T^A S_Q(p)] \rangle. \quad (6)$$

The leading-order terms of this HQE are obtained for the heavy-quark propagators  $S_Q^{(1)}$  containing  $\tilde{A}_\mu^{(1)}$  and  $S_Q^{(2)}$  with leading-order background fields  $\tilde{A}_\mu^{(0)}$ :

$$\langle \bar{q} \Gamma T^A q \bar{Q} \Gamma' T^A Q \rangle = -i \int \frac{d^4 p}{(2\pi)^4} \langle \bar{q} \Gamma T^A q \text{Tr}_{c,D} [\Gamma' T^A (S_Q^{(1)}(p) + S_Q^{(2)}(p) + \dots)] \rangle \quad (7)$$

$$= \langle \bar{q} \Gamma T^A q \bar{Q} \Gamma' T^A Q \rangle^{(0)} + \langle \bar{q} \Gamma T^A q \bar{Q} \Gamma' T^A Q \rangle^{(1)} + \dots \quad (8)$$

$$= \text{Diagram 1} + \text{Diagram 2} + \dots$$

Evaluation of the first term of the expansion (8) for the complete list of two-flavour four-quark condensates in [17] gives three non-zero results:

$$\langle \bar{q} \gamma^\nu t^A q \bar{Q} \gamma_\nu t^A Q \rangle^{(0)} = -\frac{2}{3} \frac{g^2}{(4\pi)^2} \left( \log \frac{\mu^2}{m_Q^2} + \frac{1}{2} \right) \langle \bar{q} \gamma^\nu t^A q \sum_f \bar{q}_f \gamma_\nu t^A q_f \rangle, \quad (9)$$

$$\langle \bar{q} \psi t^A q \bar{Q} \psi t^A Q \rangle^{(0)} = -\frac{2}{3} \frac{g^2}{(4\pi)^2} \left( \log \frac{\mu^2}{m_Q^2} + \frac{2}{3} \right) \langle \bar{q} \psi t^A q \sum_f \bar{q}_f \psi t^A q_f \rangle, \quad (10)$$

$$\langle \bar{q} t^A q \bar{Q} \psi t^A Q \rangle^{(0)} = -\frac{4}{3} \frac{g^2}{(4\pi)^2} \left( \log \frac{\mu^2}{m_Q^2} - \frac{1}{8} \right) \langle \bar{q} t^A q \sum_f \bar{q}_f \psi t^A q_f \rangle, \quad (11)$$

where logarithmic singularities are calculated in the  $\overline{\text{MS}}$  scheme,  $\mu$  is the renormalization scale, and  $t^A = T^A$  for  $A = 1, \dots, 8$ . The non-zero contributions for the second term of (8) read

$$\langle \bar{q} q \bar{Q} Q \rangle^{(1)} = -\frac{1}{3} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle \bar{q} q G_{\mu\nu}^A G^{A\mu\nu} \rangle, \quad (12)$$

$$\langle \bar{q} t^A q \bar{Q} t^A Q \rangle^{(1)} = -\frac{1}{6} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle d^{ABC} \bar{q} t^A q G_{\mu\nu}^B G^{C\mu\nu} \rangle, \quad (13)$$

$$\langle \bar{q} \gamma_5 q \bar{Q} \gamma_5 Q \rangle^{(1)} = -\frac{1}{4} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle i \bar{q} \gamma_5 q G_{\mu\nu}^A G_{\alpha\beta}^A \epsilon^{\mu\nu\alpha\beta} \rangle, \quad (14)$$

$$\langle \bar{q} \gamma_5 t^A q \bar{Q} \gamma_5 t^A Q \rangle^{(1)} = -\frac{1}{8} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle i d^{ABC} \bar{q} \gamma_5 t^A q G_{\mu\nu}^B G_{\alpha\beta}^C \epsilon^{\mu\nu\alpha\beta} \rangle, \quad (15)$$

$$\langle \bar{q} \psi q \bar{Q} Q \rangle^{(1)} = -\frac{1}{3} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle \bar{q} \psi q G_{\mu\nu}^A G^{A\mu\nu} \rangle, \quad (16)$$

$$\langle \bar{q} \psi t^A q \bar{Q} t^A Q \rangle^{(1)} = -\frac{1}{6} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle d^{ABC} \bar{q} \psi t^A q G_{\mu\nu}^B G^{C\mu\nu} \rangle, \quad (17)$$

$$\langle \bar{q} \sigma_{\mu\nu} t^A q \bar{Q} \sigma^{\mu\nu} t^A Q \rangle^{(1)} = -\frac{5}{6} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle f^{ABC} \bar{q} \sigma_{\mu\nu} t^A q G_{\mu\nu}^B G_{\lambda\tau}^C \epsilon^{\lambda\mu\nu\tau} \rangle, \quad (18)$$

$$\langle \bar{q} \sigma_{\mu\nu} t^A q \bar{Q} \sigma_{\alpha\beta} t^A Q g^{\mu\alpha} v^\nu v^\beta \rangle^{(1)} = -\frac{5}{3} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle f^{ABC} \bar{q} \sigma_{\mu\nu} t^A q G_{\mu\nu}^B G_{\alpha\beta}^C v^\mu v_\beta \rangle, \quad (19)$$

$$\langle \bar{q} \gamma_5 \gamma_\lambda t^A q \bar{Q} \sigma_{\mu\nu} t^A Q \epsilon^{\mu\nu\lambda\tau} v_\tau \rangle^{(1)} = -\frac{5}{6} \frac{g^2}{(4\pi)^2} \frac{1}{m_Q} \langle f^{ABC} \bar{q} \gamma_5 \gamma_\lambda t^A q G_{\mu\nu}^B G_{\alpha\beta}^C \epsilon^{\mu\nu\alpha\beta} v_\tau \rangle, \quad (20)$$

where  $f^{ABC}$  is the anti-symmetric structure constant of the color group and the corresponding symmetric object  $d^{ABC}$  is defined by the anti-commutator  $\{t^A, t^B\} = \delta^{AB}/4 + d^{ABC}t^C$ .

## 4 Summary and Conclusions

The extension of the OPE for QCD sum rules of  $\bar{q}Q$  and  $\bar{Q}q$  mesons by four-quark condensates to mass dimension 6 yields heavy-light condensate contributions requiring HQE in a nuclear medium. The necessary steps to generalize the vacuum HQE [12] to cover in-medium situations are described and a general formula for the HQE of in-medium heavy-light four-quark condensates is presented. The two leading-order terms of this expansion for the complete list of two-flavour four-quark condensates [17] have been evaluated. In leading-order the results contain known condensate structures, thus, reducing the number of condensates entering the sum rule evaluation of mesons composed of a heavy and a light quark. It can be seen that the series does not exhibit a simple expansion in  $1/m_Q$ , not even in vacuum. Therefore, the lowest order terms are not suppressed by inverse powers of  $m_Q$  as for  $\langle\bar{Q}Q\rangle$ , challenging the omission of heavy-light four-quark condensates, as often done in previous sum rule analyses.

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## References

- [1] L. Tolos, C. Garcia-Recio, J. Nieves, Phys. Rev. C **80**, 065202 (2009)
- [2] D. Blaschke, P. Costa, Y.L. Kalinovsky, Phys. Rev. D **85**, 034005 (2012)
- [3] S. Yasui, K. Sudoh, Phys. Rev. C **87**, 015202 (2013)
- [4] M. He, R.J. Fries, R. Rapp, Phys. Rev. Lett. **110**, 112301 (2013)
- [5] T. Hilger, R. Thomas, B. Kämpfer, Phys. Rev. C **79**, 025202 (2009)
- [6] Z.G. Wang, T. Huang, Phys. Rev. C **84**, 048201 (2011)
- [7] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **147**, 385 (1979)
- [8] L.J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rept. **127**, 1 (1985)
- [9] S. Narison, *QCD as a Theory of Hadrons – From Partons to Confinement* (Cambridge University Press, 2004)
- [10] K. Suzuki, P. Gubler, K. Morita, M. Oka, Nucl. Phys. A **897**, 28 (2013)
- [11] S. Narison, Nucl. Phys. B **718**, 1321 (2013)
- [12] S.C. Generalis, D.J. Broadhurst, Phys. Lett. B **139**, 85 (1984)
- [13] T. Hilger, B. Kämpfer, S. Leupold, Phys. Rev. C **84**, 045202 (2011)
- [14] S. Zschocke, T. Hilger, B. Kämpfer, Eur. Phys. J. A **47**, 151 (2011)
- [15] D. Bagán, J.I. Latorre, P. Pascual, Z. Phys. C **32**, 43 (1986)
- [16] E. Bagán, J.I. Latorre, P. Pascual, R. Tarrach, Nucl. Phys. B **254**, 555 (1985)
- [17] R. Thomas, T. Hilger, B. Kämpfer, Nucl. Phys. A **795**, 19 (2007)